Diagonals and hypergeometric functions

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The diagonal of a multivariate power series $g = \sum_{i_1,\ldots,i_n} g_{i_1,\ldots,i_n} x_1^{i_1} \cdots x_n^{i_n} \in \mathbb{Q}[\![x_1,\ldots,x_n]\!]$ is defined as the univariate series $\mathrm{Diag}(g) = \sum_{j \geq 0} g_{j,\ldots,j} t^j \in \mathbb{Q}[\![t]\!]$. Diagonals not only have intriguing intrinsic properties, but also play an important role in combinatorics, in the study of special functions and even in physics. Although much is known about them, fundamental conjectures surprisingly remain open. In this talk we are interested in the following unsolved (dual) questions:

(i) What are the rational series $g(x_1,\ldots,x_n)$ whose diagonal is a power series of a sequence following a first order linear recursion, i.e $\mathrm{Diag}(g)$ is a generalized hypergeometric function $_pF_q$?

(ii) What are the hypergeometric sequences $(a_j)_{j \geq 0}$ whose generating functions $\sum_{j \geq 0} a_j t^j$ can be written as diagonals of rational power series?

While (i) is known to be algorithmically decidable, the status of (ii) is still famously unclear: for example, Christol's old but still open problem is to decide whether the function $_3F_2([1/9, 4/9, 5/9], [1/3, 1], t)$ can be written as the diagonal of a rational series.

Recently, Abdelaziz, Koutschan and Maillard [1], and shortly later Bostan and the speaker [2] achieved progress on the questions (i) and (ii). The talk will explain the main ideas, methods and difficulties of the approaches. It will provide more insight in the algorithmic and experimental nuances of the main results.

Keywords
Hypergeometric functions, Diagonals, Christol's conjecture

References
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