ICMS Session 8: Symbolic Integration

1. **Clemens G. Raab**: *Computer algebra tools for integrals*
2. John May, **Austin Roche**: *A Discussion of the Practical Issues of Computing Integrals in Maple*
3. **David Jeffrey**, Albert D. Rich: *Recent Developments in the RUBI Integration Project*

4. **James H. Davenport**: *Complexity of Integration, Special Values, and Recent Developments*
5. **Waldek Hebisch**: *Integration in terms of exponential integrals and incomplete gamma functions*
6. **Lin Jiu**: *The Method of Brackets*

Symbolic Integration

Evaluating integrals symbolically is a major application of computer algebra systems.

- Testsuite by Wester: considerable portion of examples were integrals.
- Example: can we trust?
- Education (e.g., Wolfram|Alpha)

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Integration comes in two different flavors:

1. Indefinite integrals, e.g., $\int x^2 e^x \,\mathrm{d}x = (x^2 - 2x + 2)e^x$
2. Definite integrals, e.g., $\int_{-\infty}^{\infty} e^{-x^2} \,\mathrm{d}x = \sqrt{\pi}$

In[123]:=
```
Integrate[Sqrt[1 - Sin[x]^2], {x, 0, z}]
```

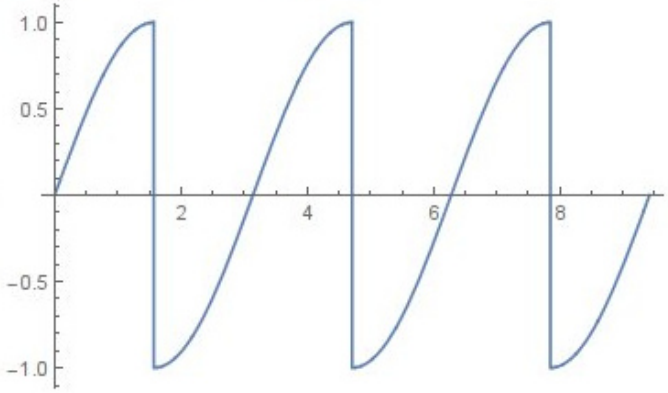
Out[123]= $\sqrt{\text{Cos}[z]^2}\ \text{Tan}[z]$

In[123]:=
```
Integrate[Sqrt[1 - Sin[x]^2], {x, 0, z}]
```

Out[123]= $\sqrt{\text{Cos}[z]^2}\ \text{Tan}[z]$

In[124]:=
```
Plot[%, {z, 0, 3 Pi}]
```

Out[124]=

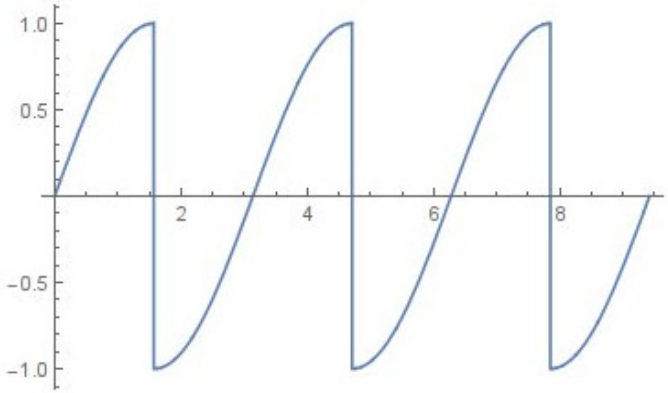


In[123]:=
```
Integrate[Sqrt[1 - Sin[x]^2], {x, 0, z}]
```

Out[123]= $\sqrt{\text{Cos}[z]^2}\ \text{Tan}[z]$

In[124]:=
```
Plot[%, {z, 0, 3 Pi}]
```

Out[124]=



In[125]:=
```
Integrate[Cos[x], {x, 0, z}]
```

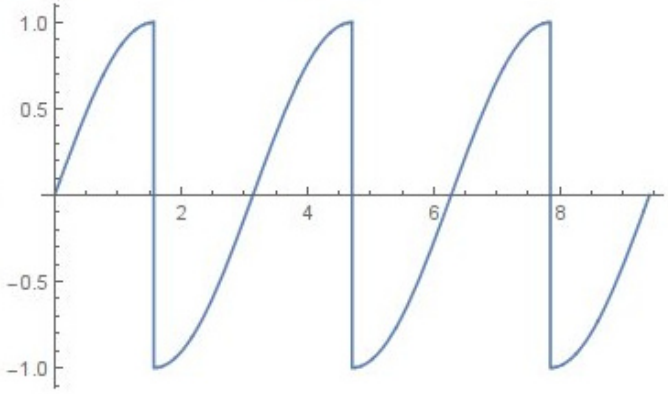
Out[125]= `Sin[z]`

In[123]:=
```
Integrate[Sqrt[1 - Sin[x]^2], {x, 0, z}]
```

Out[123]=
$$\sqrt{\text{Cos}[z]^2}\ \text{Tan}[z]$$

In[124]:=
```
Plot[%, {z, 0, 3 Pi}]
```

Out[124]=



In[125]:=
```
Integrate[Cos[x], {x, 0, z}]
```

Out[125]=
$$\text{Sin}[z]$$

In[126]:=
```
Integrate[Abs[Cos[x]], {x, 0, z}]
```

Out[126]=
$$\text{ConditionalExpression}\left[2\,\text{IntegerPart}\left[\frac{z}{\pi}\right] + \begin{cases} -1 & \text{FractionalPart}\left[\frac{z}{\pi}\right] == -\frac{1}{2} \\ -2 - \text{Sin}\left[\pi\,\text{FractionalPart}\left[\frac{z}{\pi}\right]\right] & \text{FractionalPart}\left[\frac{z}{\pi}\right] < -\frac{1}{2} \\ 2 - \text{Sin}\left[\pi\,\text{FractionalPart}\left[\frac{z}{\pi}\right]\right] & \text{FractionalPart}\left[\frac{z}{\pi}\right] > \frac{1}{2} \\ \text{Sin}\left[\pi\,\text{FractionalPart}\left[\frac{z}{\pi}\right]\right] & \text{True} \end{cases}\right.$$

Symbolic Integration

Different approaches:

- Database of formulas
- Integration in finite terms
  - Classical example: integration of rational functions (partial fraction decomposition, Hermite reduction, Rothstein-Trager resultant)
  - Extension to algebraic functions, Liouvillian functions (Risch, Davenport, Trager, Bronstein, ...)
  - Differential fields
- Creative telescoping
  - Derive an ODE or recurrence for a definite integral, by "differentiating under the integral sign".
  - Holonomic functions (Zeilberger, Takayama, Chyzak, ...)
  - Differential fields (Raab)
  - Reduction-based algorithms
- Many more special purpose algorithms ("method of brackets", special functions, etc.)

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